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The Theatre of Spin Glass: some ongoing debates and broad applications of spin glass physics

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Abstract

The theories describing spin glass (a magnetic state originally observed in manganese alloys) have been used with exceptional consistency to advance fields as diverse as neuroscience, observational biology, and economics. This is because manifold research objects of those communities resemble the randomness, competition, and frustration of interacting spins typical for the spin glass. By comparing its adequate and possibly inadequate interdisciplinary translations, this review addresses the problems that researchers may face when applying spin glass to their fields.

Centrally, the juxtaposition of two disparate spin glass' descriptions (continuous-in-nature Replica Symmetry Breaking method with discrete droplet-scaling) applied in graph theory, flocking, and financial modelling problems yielded a set of requirements ensuring a valid translation.

Thus, the value of this review is expected to be twofold.

1. In general, the translational requirements are hoped to add a new tool for those using interdisciplinary methods in science.
2. In particular, these requirements shall gauge how spin glass may influence the research in quantum neural networks. The analysis focuses on the "quantumness of brain" to reexamine the views presented, for instance, by Schuld *et al.* [1] in the context of new advancements in relevant fields (e.g., photonics).

With the broader outlook, the following discussions encourage the specialist in, e.g., quantum computing to use spin glass models more often. Upgrading devices to the quantum world is projected to bring about an unprecedented-scale revolution in terms of computational power and speed. It could be topped with an improved understanding of brain processes.

Conventions

This review uses the following typographic conventions. **New terms** are in bold. Their *definitions* are in italics. E m p h a s e s by the author are spaced-out.

Abbreviations

BM – Boltzmann Machine

EA – Spin Glass model by Edwards-Anderson

N&N – Spin Glass model by Nishimori-Nonomura

NN – Neural Network

OP – Order Parameter

QA – Quantum Annealing

QC – Quantum Computing

QM – Quantum Mechanics

QMi – Quantum Mind

QNN – Quantum Neural Network

RSB – Replica Symmetry Breaking

SG – Spin Glass

SK – Spin Glass model by Sherrington-Kirkpatrick

TSP – Travelling Salesman Problem

1 The overture to research in spin glass

The way clever actors imprint their styles on the variety of characters is reminiscent of the ways the framework of spin glass (SG) influences numerous areas of research. Combinatorial mathematicians, solid-state and laser-physicists, biologists, neurologists, machine learning and materials scientists have advanced their communities prodigiously by comparing systems of their interest to the three SG-phenomena [2]. These are **randomness, competition, and frustration of variables.** They define SG.

1.1 The construction of the problem

Although randomness, competition, and frustration are broadly-established in literature, scientists still look for their more accurate equivalents in the semantics of their communities. Because well-grounded correspondences between SG and similarly complex systems have led to extraordinary advancements like those in flight-scheduling optimisation [2, 3], some fields now try to adapt this framework to their research questions. This task has not been scrutinised enough in the context of SG [4]. Worse still, some communities have consequently used limited or poorly constructed analogies.

For instance, see Fig. 2: a study on a flock of birds by Bialek *et al.* [5] relies on assumptions that make the flock similar to SG but may not fully reflect the biological reality – more in section 3.2. In Sophocles' language, the authors committed a **scientific hamartia**[1] limiting the validity of their arguments.

Such interpretational problems are of importance. According to Google Scholar, about thirty thousand publications mentioned "spin glass" in 2021 alone.

To solve the common issues of the interested communities is to present randomness, competition, and frustration in SG first, and then explore their translations.

[1]**hamartia** [ˌhɑː.mɑːˈtiː.ə] *noun* – (in Greek tragedy) the protagonist's misdiagnosis of their situation leading (...) ultimately to a catastrophe. Adapted from the *Cambridge Dictionary.*

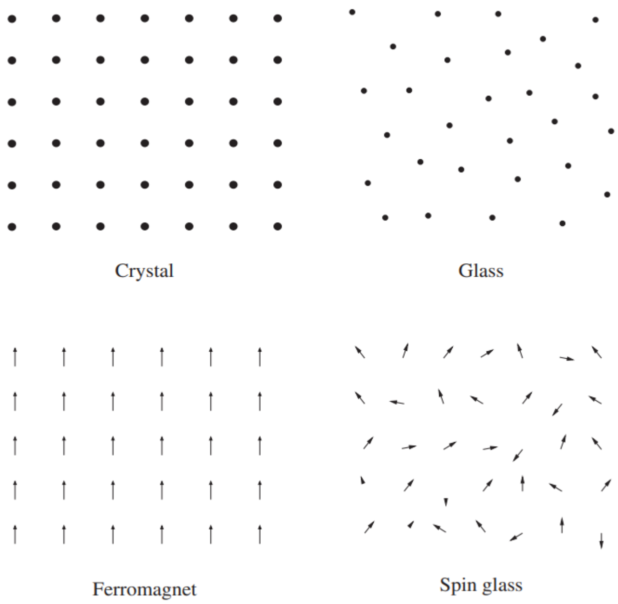


Figure 1: By contrast to crystal (top left), the constituent atoms of glass (top right) are irregularly-spaced on a lattice. By extension to crystal, the constituent atoms of ferromagnet (bottom left) have a *magnetic moment* called **spin**. These two phases combined give a spin glass (bottom right). From [6].

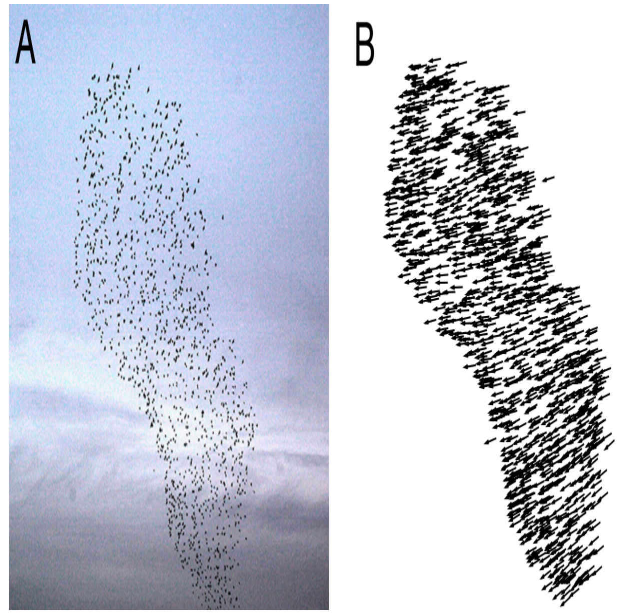


Figure 2: Picture B represents the velocities of almost 1300 birds on picture A. Note the seeming similarity of B and spin glass in Fig. 1 prompted the authors of [5] to re-use its framework; some assumptions that allowed for this comparison were unjustified, nonetheless. From [5].

1.2 Definition and beginnings of spin glass

Firstly, a SG is a set of **randomly-located spins**. They may *point in random directions in space* – Fig. 1. On another level, be it location in isolation from other spins, or *aggregation* in so-called **clusters**, spins may randomly adopt both kinds of positioning.

Secondly, the spins *interact with neighbours either ferromagnetically or antiferromagnetically.* This local **competition** remains unresolved globally: *on average there is an equal number of both interactions* which makes the disorder **quenched**.

Finally, the SG spins are **frustrated**. Though *their orientation minimises the energy of magnetic interaction with most neighbours, it does not do so for all of them.* One of the greatest SG-researchers, noble Prof. Parisi, described it metaphorically [7]; in paraphrase, like the supporters of the Capulets (associated with ferromagnetically interacting spins) and the Montagues (antiferromagnetic ones), each spin knows how it should generally behave but it does not prevent it from having conflicting sentiments towards some spins.

Theatre aside: the beginnings of SG relate to then-inexplicable experiments in metals. In the 1950s, groups of Owen, Kip, and others, investigated how free electrons' exchange interaction with magnetic ions influenced their resonance behaviour [8]. In detail, they diluted manganese into nonmagnetic copper.

Of much relevance to explaining the resonant behaviour were hoped to be the measurements of specific heat and magnetic susceptibility in [9]. Unexpectedly, the Mn-Cu-alloy had five-fold greater heat capacity than the reference copper. This behaviour correlated with the magnetic properties of the Mn-Cu-alloy evidenced by the disagreement of susceptibility data and Curie law, which governs magnetism in solids.

As shown later, the magnetic and thermal behaviour could be approximately modelled by the Ruderman-Kittel-Kasuya-Yosida interaction,

$$\begin{aligned} J(r_{ij}) &= \text{Interaction depending on} \binom{\text{the distance between}}{\text{the } i^{\text{th}} \text{ and } j^{\text{th}} \text{ spins}} = \\ &= \frac{\text{Interaction constant}}{\binom{\text{the distance between}}{\text{the } i^{\text{th}} \text{ and } j^{\text{th}} \text{ spins}}^{\text{cubed}}} \cdot \left(\text{cosine of } \left[2 \cdot \binom{\text{Fermi momentum of}}{\text{conduction electrons}} \cdot \binom{\text{the distance between}}{\text{the } i^{\text{th}} \text{ and } j^{\text{th}} \text{ spins}}\right]\right) = \\ &= \frac{J_0}{r_{ij}^3} \cos(2k_F r_{ij}) \end{aligned} \tag{1}$$

which did not explain the resonant behaviour, nevertheless. It took twenty years to fully explain this experiment – see 2.1.

1.3 The repertoire

This review shall build upon ??-1.2 in three ways. In

- Act I: the approaches to modelling SGs will be re-examined, emphasising the opposite-in-nature replica and droplet-scaling methods,
- Act II: a set of translation requirements will be inferred from the applications of SG in graph theory, biology, and finance,

- Act III: an emerging field of quantum neural networks (QNNs) will be showcased in terms of the influence SG has had and can have on it.

Along the way, examples of the **scientific hamartia** (considered as a *faulty/incomplete use of the SG-framework to a particular matter*) will be pointed out.

It has not escaped the author's attention that the review may elucidate the paradigms of interdisciplinary science (Fig. 3). For this purpose, let **interdisciplinarity** be the *use of system's properties preserved upon the translation from one setting to another.*

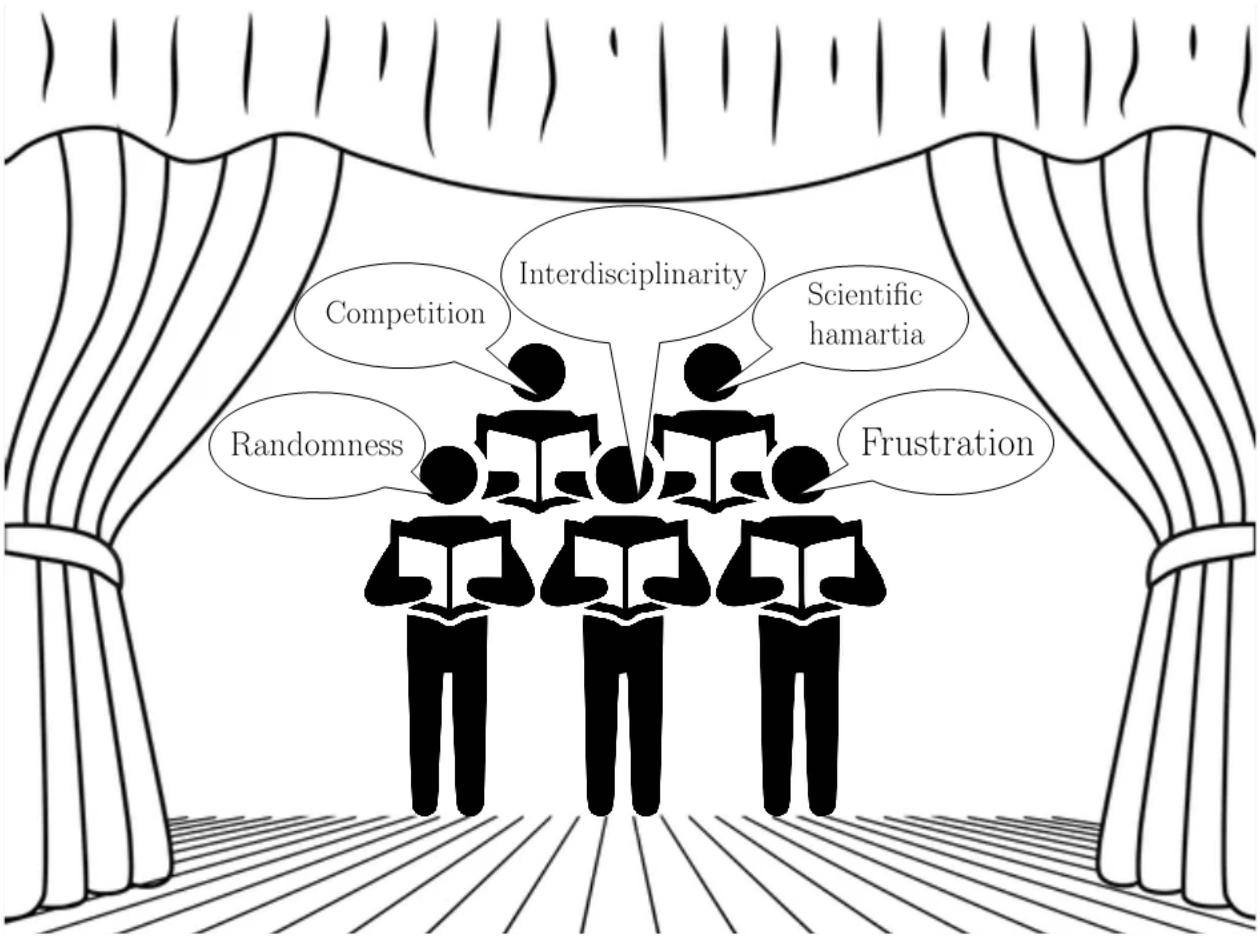


Figure 3: A graphical summary of the introduction – welcome to the "Theatre of Spin Glass". The choir announces the themes of the play like competition of spins and scientific hamartia.

2 Act I: The droplet-scaling method strikes back – a duel with the replica and others methods

What sustains SG as a vibrant area of research is not only its powerful versatility beyond materials but primarily the debates due to the disparate models. Conversely to the state-of-the-art in unification of General Relativity and Quantum Mechanics (QM), SG has already got two sets of agreeing, opposite-in-nature theories: continuous-order-parameter (e.g., Replica Symmetry Breaking (RSB)), and discrete-order-parameter ones (notably droplet-scaling).

Both approaches were to explain frustration in more detail through improving **order parameters** (OP). These *functions measure the order across the boundaries in phase-transition systems*. Case-in-point, such a parameter is the difference in densities. The OP of water evaporation is negative because water is denser than steam. OPs involving, e.g., entropy, magnetisation, etc., are also useful.

For three decades, continuous-OP methods have been more successful, culminating in the appreciation from the Nobel Committee [10]. Defiantly, this section will show how the discrete methods have recently been developed computationally, destabilising agreements.

2.1 Continuous-order-parameter methods

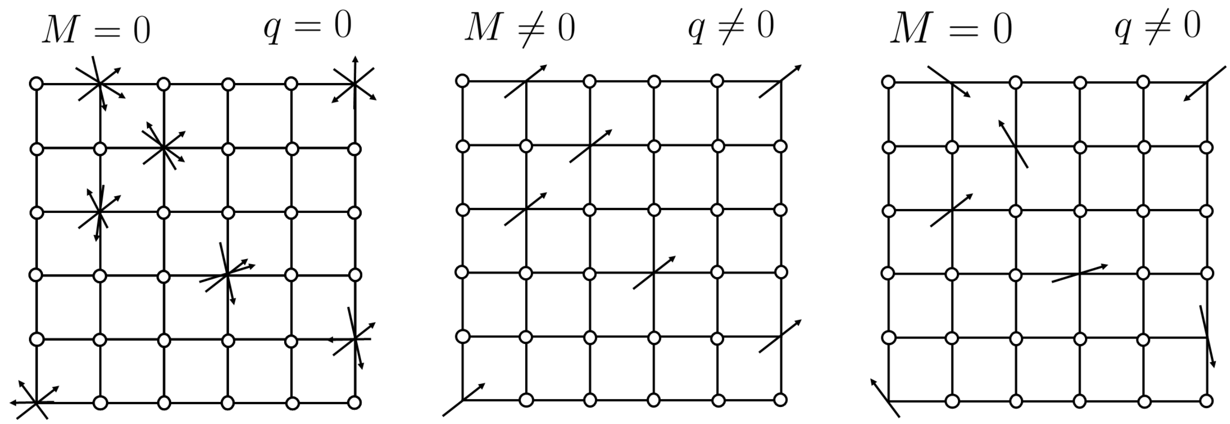
1975 saw Edwards and Anderson making a breakthrough in the understanding of frustration [11]. They suggested that

$$\mathcal{H}_{\mathrm{EA}} = \sum_{\substack{\text{nearest}\\ \text{neighbours}}} \left(\substack{\text{exchange interaction parameter}\\ \text{for the i}^{\text{th}}\text{ and j}^{\text{th}}\text{ spins}}\right) \cdot \overrightarrow{\text{spin}_i} \cdot \overrightarrow{\text{spin}_j} = \sum_{\langle ij \rangle} J_{ij} \overrightarrow{S_i} \cdot \overrightarrow{S_j} \tag{2}$$

describes the interaction between regularly-spaced neighbouring classical vectors $\overrightarrow{S}$. Despite idealising disordered SG in this fashion, they got an accurate OP

$$q_{\mathrm{EA}} = \left(\substack{\text{configurational}\\ \text{average}}\right) \text{ of } \left(\substack{\text{the thermal}\\ \text{average of spins}}\right)^{\text{squared}} = \left[\langle S_i \rangle_T^2\right]_{\mathrm{Av}} \tag{3}$$

illustrated in Fig. 4.



(a) Paramagnet (b) Ferromagnet (c) Spin glass

Figure 4: Differences between a paramagnet, ferromagnet and spin glass. Here, $M = \frac{1}{N}\sum_i m_i$, where m_i is the magnetisation at site i and N is the number of spins. Even though derived through a continuous method, the order parameter simplifies to $q_{\mathrm{EA}} = \frac{1}{N}\sum_i m_i^2$. For spin glass, $q \neq 0$. Adapted from [2].

The EA-model continued to spark discussions for three reasons.

Firstly, Sherrington and Kirkpatrick pointed out that had Edwards and Anderson been even cleverer [12], they would have used the 1971 observation by Stanley [13]. Expressly, the mean-field theory of ferromagnetism becomes exact when every spin interacts with e v e r y s p i n instead of closest neighbours only. Thus, $\mathcal{H}_{\mathrm{EA}}$ was extended to

$$\mathcal{H}_{\mathrm{SK}} = -\begin{pmatrix}\text{Edwards}-\text{Anderson}\\ \text{Hamiltonian}\\ \text{summed over all spins}\end{pmatrix} - \begin{pmatrix}\text{uniform}\\ \text{external}\\ \text{field}\end{pmatrix} \cdot \sum_i \begin{matrix}\text{modulus}\\ \text{of } \overrightarrow{\text{spin}_i}\end{matrix} = -\sum_{(ij)} J_{ij}\overrightarrow{S_i}\cdot\overrightarrow{S_j} - h\sum_i \left|\overrightarrow{S_i}\right|, \quad (4)$$

resulting in an u n s t a b l e s o l u t i o n, implying negative entropy.

Secondly, since the exchange interaction J_{ij} is derived from $p\left(J_{ij}\right) = \frac{1}{\sqrt{2\pi J^2}}e^{-\frac{J_{ij}}{2J^2}}$, it was debated which macroscopic properties of SG are dependent on a particular realisation of this distribution. Further discussion in 3.2.

Thirdly, the Edwards-Anderson paper introduced the **replica trick**. It *averages some properties of the system, using n copies (replicas) thereof*. However, this concept (e.g., of approximating the average of the partition function $\overline{\ln Z}$ by disorder average $\overline{Z^n}$, where $n \in \mathbb{N}$ approaches zero) was not materialised until Parisi [14]. He realised that the accurate description requires i n f i n i t y of OPs with hierarchical structure – Fig. 5.

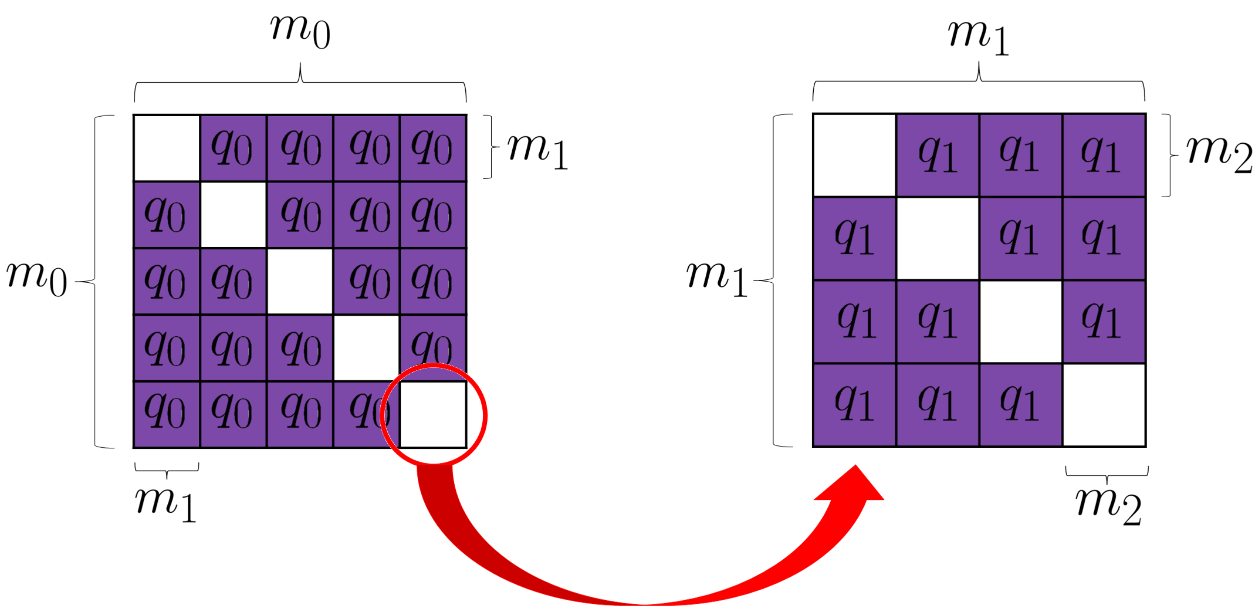


Figure 5: Replica trick: (Left) Divide $m_0 \times m_0$ matrix into $m_1 \times m_1$ boxes. Extending the SK-model, assign q_0 only to off-diagonals. (Right) Divide the diagonal boxes into $m_2 \times m_2$ ones. Assign q_1 to off-diagonals. Repeat for $m_0 \geqslant m_1 \geqslant m_2 \geqslant ... \geqslant m_d \geqslant 1$. Zero the diagonal. Now, $\{q_0 = q(m_0), q_1 = q(m_1), ..., q_d = q(m_d)\}$ describes the order parameter (continuous for $m_0 \to 0,\ d \to \infty$). Adapted from [2].

What is innovative about Parisi's thinking? Although the starting matrix in Fig. 5 is collapsed from $m_0 \times m_0$ to 0×0, it is endowed with infinite structure. This is hard to visualise, but mercifully, what emerges from the OP set is a continuous function $q(x)$ on $[0, 1]$. If replica symmetry is unbroken, $q(x)$ is a constant; otherwise, it has a complicated form.

This discovery noticeably impacted the community. Remarkably, Parisi elucidated the instability of the SK-model with **breaking** of the replica symmetries; namely, *how the system is influenced by small fluctuations.* Replicas also illuminated self-averaging and ultrametricity[2].

After the landmark publication with Mézard [7], Parisi revised his views twice.

First in 2000, replying to criticism from Newman and Stein that only some structures for replicas and their overlaps are allowable by temperature and dimensionality [15, 16], Parisi *et al.* proposed a number of correlation functions to solve it [17]. This work also showed the numerical calculation of the phase-transition temperature for SG.

[2]For details, see an approachable review [2] and 3.1.

Later, having established the credibility of his theory, Parisi encouraged his students to extend it to random regular graphs [18].

2.2 Old adversaries: Discrete-order-parameter methods

Not only were Parisi's followers doubtful, but also the advocates of discrete-OP methods have given overturning his ideas a go. Were they successful nemeses?

Two 'founding' publications for both sub-communities – both from 1987 – will serve as a quantitative answer. More than 6000 citations of [7] by Parisi *et al.* vs fewer than 600 of [19] by Moore *et al.* as of writing this speaks volumes. It is important to examine the droplet-scaling picture, nevertheless, because its development was mainly suppressed by the limited computation power of computers until the 2010s.

Even with this context, it remains counter-intuitive that the theory – explaining low-temperature SG with a finite dimensionality almost ten years before Parisi did it in 1993 [20] – was sidelined. The first papers by McMillan, and Fisher with Huse [21, 22] advertised it as "no 'RSB', (...) 'lack of self-averaging' [approach]" [19].

Instead of infinite range, these papers considered *the smallest possible clusters of spins in 3D on the lengthscale L smaller than the critical lengthscale L^** – so-called **droplets**. The rationale was that the static and dynamic correlation functions at long distances and times are dominated by low-energy large-droplet excitations [22]. The OP required t w o numbers, but it was applicable only in the vicinity of the critical temperature T_c.

In the 1990s, this finite-OP method could reproduce most of the results from the Replica method. Papers like [23] polemised Parisi's work, introducing the L-dependent overlap distribution

$$P(q,L) = \left(\substack{\text{configurational}\\\text{average}}\right) \text{ of } \left(\substack{\text{thermal}\\\text{average}}\right) \text{ of } \left(\substack{\text{Kronecker}\\\text{delta}\\\text{function}}\right) \text{ of } \left(\substack{\text{overlap}\\\text{/order}\\\text{parameter}} - \frac{\sum_{i=1}^{N} \text{weight}_i \cdot \text{first replica}_i \cdot \text{second replica}_i}{\sum_{i=1}^{N} \text{weight}_i}\right) = \\ = \left[\left\langle \delta\left(q - \frac{\sum_{i=1}^{N} x_i S_i^{(1)} S_i^{(2)}}{\sum_{i=1}^{N} x_i}\right)\right\rangle_T\right]_{\text{Av}} \tag{5}$$

Using this tool, authors concluded the compatibility of the two methods near $T = 0.7T_c$ and better performance of droplet-scaling for lower temperatures – Fig. 6. The paper remarks at last about the possibility of extending the model to six dimensions.

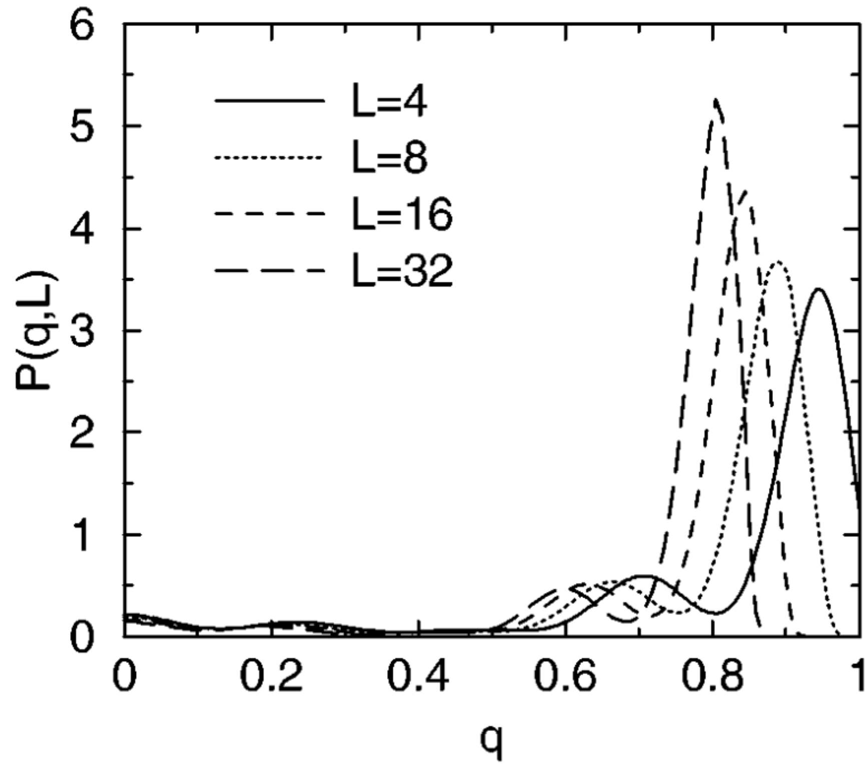


Figure 6: While numerical data around $T = 0.7T_c$ is compatible with RSB, data for lower temperatures favours the droplet picture. See how $P(q, L)$ $(T = 0.38T_c)$ decreases with increasing system size for small q. Then, the area under the subsidiary bumps decreases, and the area under the main peak increases. From [23].

The field reinvigorated again due to several publications using more efficient computers in the 2010s – e.g., [24] and [25]. By considering diluted SG interacting on a d-dimensional hypercubic lattice for small d, the former supported the droplet-scaling further in the context of determination of pure states structure. Conversely, the latter claimed that structure to be nontrivial (i.e., not compliant with the droplet-scaling) even above the upper critical range.

Amidst these results, Moore revisited his ideas in 2021 [26], still advocating the superiority of his child to that of the noble Italian. This brings us to the assessment of the future usefulness of the two methods.

2.3 New skirmishes – Future uses

There are a few areas where scientists are not sure which theory will be more useful. The current debates (Fig. 7) concern:

1. which quantum SG are physically realisable,
2. what the nature of the SG phase transition is,
3. what influence SG can have on complexity studies still.

Ever since EA, scientists have debated if their models reflect reality. With the models' gradual refinement in terms of the quantum phenomena, these discussions extended. The RSB has had more influence on them, e.g., [27], but droplet-scaling is also used by virtue of being a discrete theory [28].

Likewise, the phase-transition problem has been described in publications from both side of the aisle, e.g., [29] vs [28]

An attempt of answer to the last question will be presented in Act III discussing QNNs.

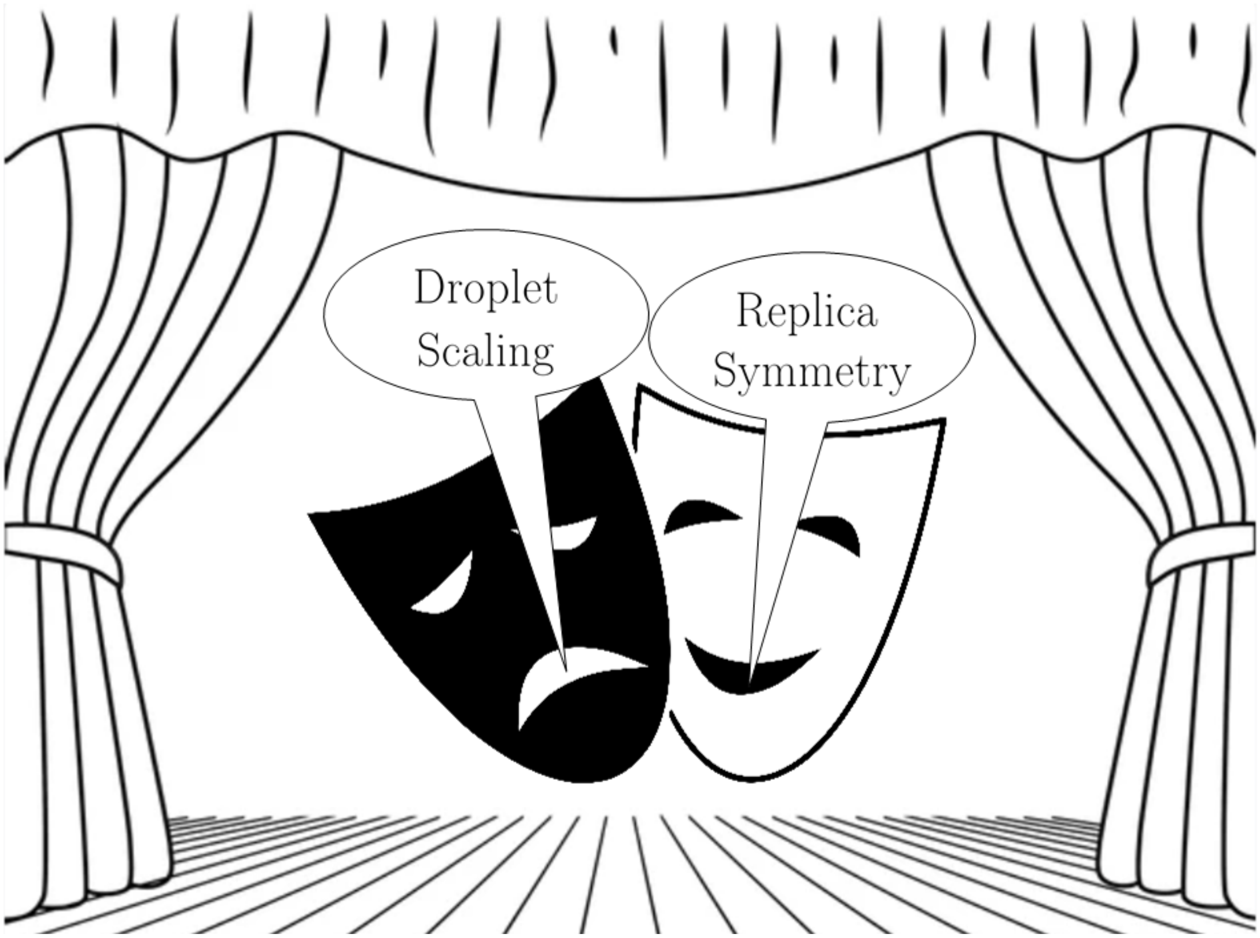


Figure 7: The way the masks of tragedy and comedy symbolised the two leading themes of Greek theatres, they summarise that there are two leading theories of SG – Replica Symmetry Breaking and droplet-scaling.

3 Act II: Spin glass' ventures beyond the materials realm

Struggle of finite and infinite, continuous and discontinuous summarises the SG-instabilities (Act I). This is especially true for its current directions like complexity studies or physical realisability.

Another area of challenge for SG-ideas are QNNs anticipated to perform more efficiently than classical NNs. Acceleration of this field by the SG-framework is of interest for mathematicians, physicists, computer scientists, and whomever using machine learning.

Therefore, this section delineates possible analogies that could be drawn, while advancing the QNNs' community. Successful and unsuccessful translations of SG into combinatorics, biology, and economics are shown.

3.1 Travelling Salesman Problem

Travelling Salesman Problem (TSP) shows how Parisi and other theatre-directors' styles were emulated. Imagine a theatrical troupe (originally, the eponymous salesman) willing to visit N cities once on a tour before going home. Which route is the shortest?

Formally, what is the shortest closed path through a given set of N points?

Let the cities be indexed by $\{1, 2, ..., N\}$. Let $P(i)$ be the index of the city visited as the i^{th} in the order. Let $l_{i,j}$ be the distance between i^{th}-ford and j^{th}-bridge. Denoting $l_{N,N+1} = l_{N,1}$, the total distance is $L = \sum_i l_{i,i+1}$. TSP is now equivalent to minimising

$$L\left(P(i) : i \in \{1, 2, ..., N\}\right) = \binom{\text{Total}}{\text{distance}} \text{ depending on } \binom{\text{order}}{\text{of visits}} = \\ = \sum_{\text{the first city}}^{\text{the last city}} \binom{\text{distance between the } i^{\text{th}} \text{ and the}}{i{+}1^{\text{th}} \text{ city in the travel plan}} \text{ given a closed path } = \\ = \sum_{i=1}^{N} l_{P(i),P(i+1)} \text{ with } P(1) = P(N+1) \quad (6)$$

The translation to SG-framework goes as follows. L is said to be a **cost function**. *It maps a particular statement of TSP onto a number representing the travel c o s t.* In mechanics, Hamiltonians are such cost functions. Therefore, L describes the energy of a particular configuration in the phase space of all travels. Even complicating the model (i.e., considering travel expenditure dependent on factors other than distance), such an expression can be minimised, yielding the optimal route.

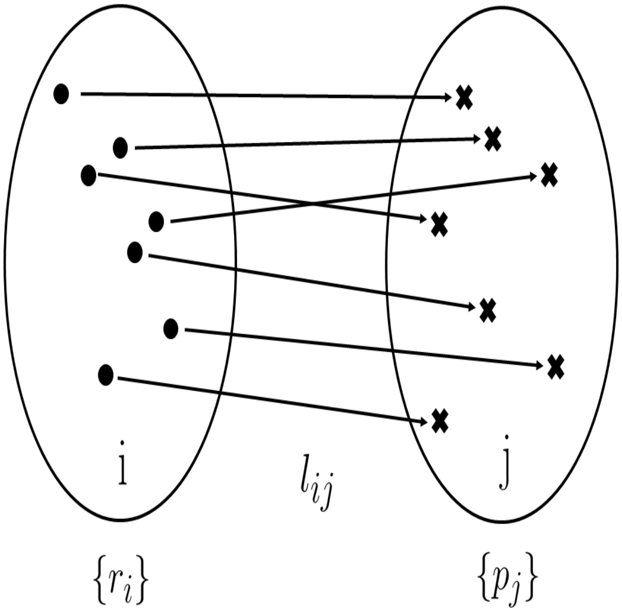


Figure 8: Bipartite Problem: For two (equinumerous) sets of d-dimensional-space points $\{r_i\}$ and $\{p_j\}$, construct bonds l_{ij} that minimise $\sum l_{ij}$. Regardless of either the points' locations random (as in TSP) or bonds (specifying a distribution $\rho(l_{ij})$) being random, the matrix l_{ij} defines an instance of the problem. Adapted from [2].

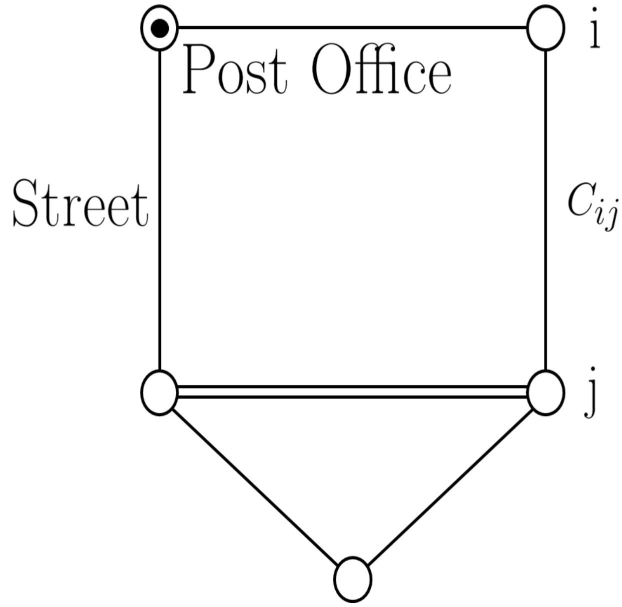


Figure 9: The Chinese Postman (Minimum Weighted Cut) problem: Starting from and returning to the post office, a postman delivers mail along the edges of a connected graph. He must go along each street at least once in either direction. What is the shortest possible route? Adapted from [2].

Despite this successful translation, why was the application of the SG-framework to TSP a disaster at first?

TSP is a **non-polynomial** problem, i.e., *the algorithm to compute L is exponential in time*, ruling out the exact computational approach [2]. In a series of poetic-in-form lectures, Kirkpatrick insisted nonetheless that the winning strategy resembles the frustration of SG in the short- (choosing the closest city) and long-range (visiting all cities once) [30]. There were two successes translation considering these characteristics.

Orland *et al.* observed that a version of TSP – Bipartite Problem (Fig. 8) – is polynomial-in-time [31]. This allowed to solve the problem with the SG-framework [32].

Chinese Postman problem exemplified the opposite direction of the knowledge transfer – Fig. 9. This graph theory problem mapped onto the SG helped to investigate the ground-state energy-level of SG [33, 34].

All in all, a successful SG-translation often requires narrowing the problem to specific contexts. Only later scientists can expand it, as it is the case for TSP (currently considered for non-symmetric costs between cities [3]).

3.2 Flocks of birds

Flocking was defined as a *collective flight of birds*, presumably minimising the drag. However, [35] reported that fish, insects, and bacteria behaved similarly. This motivates then-ongoing discussions if the phenomenon was more general. Humans transpired to flock, but recent studies challenge the accepted explanations why they do it, e.g., when walking on a bridge [36].

The author of this review sees that the deficient explanation might have been due to a faulty translation of the SG-framework to flocking. Sherrington and Kirkpatrick put in considerable effort to justify that their model resembled spins in metals, while modelling flocks with SG artificially introduced randomness to flocks. For flock-researchers assume a random distribution of velocities like in Fig. 2B best matching a snapshot like in Fig. 2A. The randomness of the distribution allows using the SG-framework.

Therefore, as alluded to in **??**, the paper by Bialek *et al.* [5] seems to the author as a candidate for scientific hamartia.

It compares the systems of SG-esque pairwise-interacting spins and flocks of starlings, motivating it by their large-scale collective behaviour. The resulting Hamiltonian is analysed to claim that local, pairwise interactions between birds are sufficient to predict the order throughout entire flocks. This would confirm the results by Parisi *et al.* [37].

Extending this result suggested that the interaction between birds must be topological rather than metrical due to the independence of the flock density [5]. The topological model applied iteratively to an increasing number of birds (starting from four) recovers the underpinning long-range patterns.

The study sparked significant interest (600 citations). What these follow-up publications seem to neglect, though, is

1. at-first inconspicuous transition from the RSB to droplet-scaling (examples: [38, 39]),
2. two assumptions reasonable for SG-systems that were not shown to coincide with the biological data (examples: [40, 41]).

The author of this review thinks that these issues look as follows in detail.

1. First, the collective large-scale behaviour is used to justify the topological nature of flocks. Clear use of the RSB method applicable to infinite SGs.
   Later, however, Bialek *et al.* applied the topological model to a few birds – as Moore did in his papers, dealing with droplets. By scaling it, the authors got the initial conclusion. Further research settling if this observation applies to SG (or to flocks exclusively) could help to unify RSB and droplet-scaling deeper.
2. (a) The approach to choose a distribution of velocities with the maximal entropy of the flock was not particularly justified – it was just extremely successful elsewhere [42]. Although this assumption was later corroborated by [43] on out-of-equilibrium flocks, the motivation was misleading: the authors might have been pre-assumptive about the flocks, and made these clusters of birds look alike SG. Besides, the maximal-entropy postulate says that the entropy of information a b o u t t h e s y s t e m (excluding the assumptions) should be maximal. This directly contrasts with what the authors did by setting the entropy o f t h e s y s t e m to maximum.
   (b) To get quantifiable results, the velocities of starlings were chosen from a probability distribution (described by a correlation matrix), which behoves to ask if any properties of the system depend on a particular realisation of that distribution (cf. EA in 2.1). [11].

> As remarked by Stein, relying on a single distribution carries the risk of losing the reproducibility of results [6]. This aspect was not considered in Bialek's and similar publications [40, 41], which could have led to differing results unless the OP of the flock had been shown to be **self-averaging**[3] [19].

In a nutshell, using the SG-framework needs to specify the method used (RSB vs droplet-scaling), the accuracy of comparison (e.g., if the system minimises entropy), and the caution with choosing distribution-functions.

3.3 Banks interconnectedness

An area of SG-application complementing the requirements for an effective translation from 3.1 and 3.2 is finance. Therein SG serves not merely as a modelling tool, but rather as a Stephen-Wolfram-type generator of pseudo-random complexity. How?

SG-framework can reproduce the complexity of financial markets disobeying the axioms of classical financial analysis (i.e., when the agents make unreasonable decisions, the **invisible hand** does not *regulate supply and demand*, and the prices are false value for money). This approach was originally coined in 1999 [44], but it had been the failure of Lehman Brothers nine years that sparked discussions about the "scientific revolution" in finance [45].

Although publications like [46] and [47] were unappreciated during the crisis[4], they finely mapped SG onto economics. For example,

$$\text{Hamiltonian} \iff \text{global optimisation function}$$

$$\text{temperature} \iff \text{economic activity}$$

$$\text{pressure} \iff \text{specific wealth}$$

By considering RSB, Krey and Preis outlined a strategy of overcoming the market crash [46] by

[3]That is *the properties of one large sample differ negligibly from an average over an ensemble of systems with different instances of quenched disorder.*

[4]Nothing shows better how niche the literature on that matter was in its infancy like the direct email-request from Krey to enthusiasts willing to discuss the paper [46].

- recapitalising the endangered financial institutions (the equivalent of applying an external magnetic field to SG), and
- deficit spending of the state (increasing the temperature).

Although these publications are doubtful to have been the motivation, these countermeasures worked, e.g., in the USA. Thus, scientists wanted to see if their predictions would be accurate when narrowed down to **systemic risks** [4]. They happen when *a single institution's failure cause collapses of other market-players* which had been observed for interconnected banks. The SG-framework was used to quantify the bank lending-dependencies. Remarkably, this approach can assess the influence of other factors than the bank-bank operations, e.g., governmental subsidies.

These SG-translations are significant for the financial firms predicting the future stock affected by unforeseeable disruptions like pandemics. Sadly, this research is normally tested empirically against real-world data and neglects the fundamental limitations of logical systems (and generally, the predictive power of logical systems – another candidate for scientific hamartia) [48]. Shafee mentioned a similar issue when he applied SG to the analysis of human decision-making algorithms three years before the 2008-recession [49].

In total, for an effective translation, the SG-framework needs to add predictive power beyond the phenomena it is employed to characterise. In finance, SG can even model the interactions of banks influenced by external factors.

3.4 Summary of requirements

How to influence another field with SG? (Fig. 10).

1. Narrow the topic to apply SG effectively (e.g., Bipartite Problem).
2. Choose the most useful SG-theory (RSB, droplet-scaling, other).
3. Check if the parameters of interest behave exactly like SG (e.g., self-averaging, entropy maximising).

4. Make your SG-application influences other problems in the field (e.g., the influence of external factors on SG-like banks).

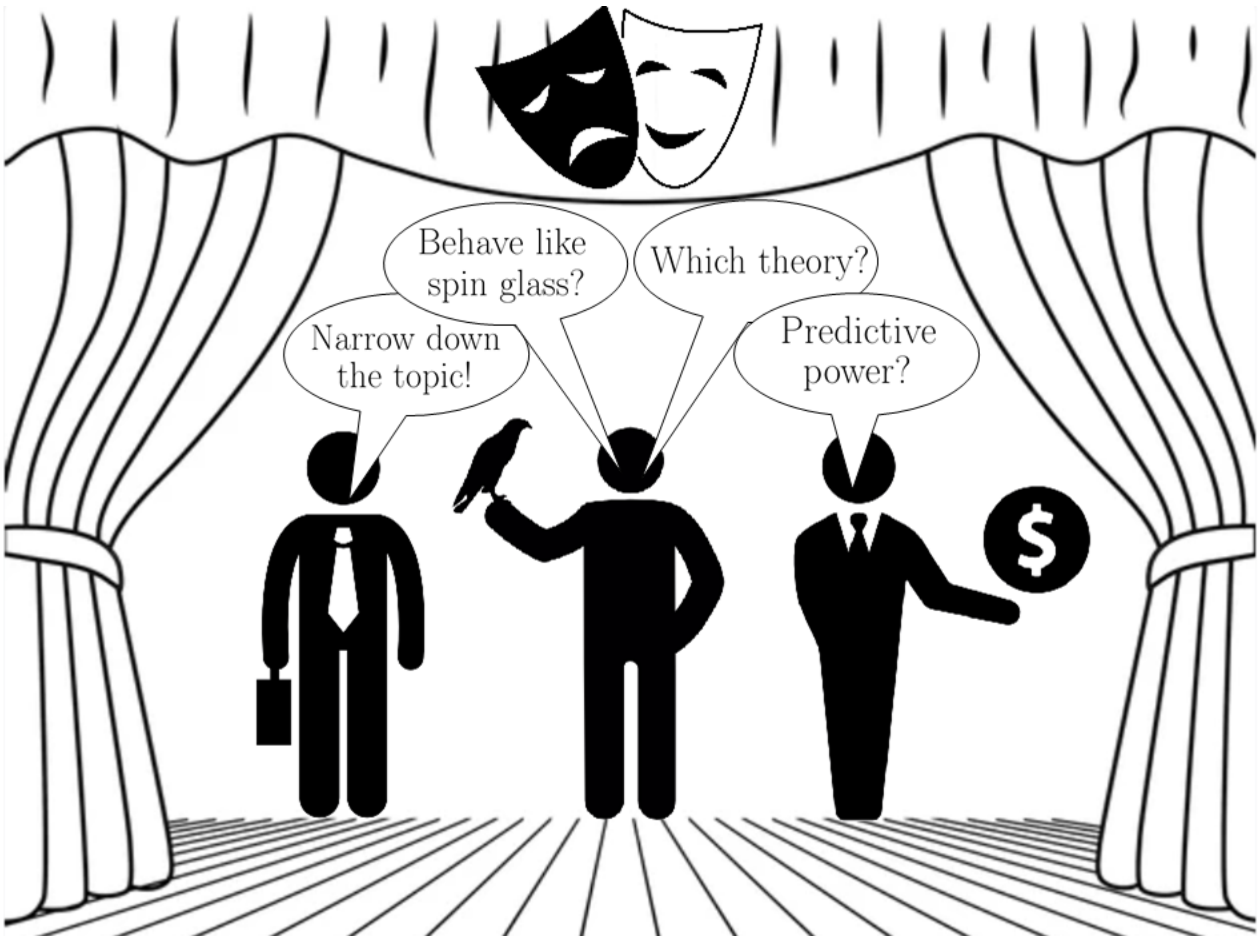


Figure 10: The lessons learnt in the review of interdisciplinary applications of SG.

4 Act III: A new battle — quantum neural networks

Quantum science's relation to SG is manifold. This review has shown the SG-models with classical spins; scientists also try to improve them by considering quantum spins [50]. Now, however, the discussion will go the opposite way and describe how SG is used to improve quantum science.

The bridge between the two is **quantum computing** (QC). It *uses superposition, interference, and entanglement, to perform calculations.* Amidst QC's development, fields like quantum machine learning try to keep up with (if not lead) these efforts. That the classical NNs, the backbone of machine learning, were based on SG is hoped to make relating QC and SG easier.

Thus, having found the translation requirements, one can assess if and how SG can accelerate the research in QNNs. This section will attempt to answer:

- How is SG relevant to QNNs?
- Have the translation requirements been met?
- What is the evidence that SG could answer open questions?

4.1 The purpose and scope of translation

Why are QNNs important foremostly? What is the endgame in pursuing this subject? One of the first answers – given by Feynman when the field was not born yet – determined the interdisciplinary translation of SG to QNNs.

In a lecture in 1982, he hoped that computers could simulate quantum systems one day [51]. Feynman saw a few complications; notably, that some algorithms may operate on negative probabilities. Nevertheless, scientists tried to use computers as he suggested.

1982 was a breakthrough-year for one more reason. A highly-cited paper by Hopfield introduced a framework of modelling assumptions for NNs. It improved the existing model by examining the information processing delay [52]. How? The paper is based on SK-model superficially but accurately, briefly noting that systems whose pairwise connections are symmetric, but random, possess locally stable states. In NNs, this stability was not interpreted as a frustrated configuration of spins, but as an equivalent of memory[5].

Hopfield concluded that content does not influence the efficiency of memorisation [52]. Whether that was true for humans was scrutinised by Kak and Chrisley who independently pursued the idea of **Quantum Mind** (QMi). In-so-doing, they defined **QNNs** as *NNs based on the principles of QM* [54, 55], and QMi as *a set of hypotheses rejecting classical mechanics as an explanation of consciousness* [56].

Although the community focused on researching artificial QNNs (unrelated to living organisms), c a t h a r s i s[6] from researching QNNs is hoped to render QMi.

4.1.1 Requirement 1: Narrowing topic

The “narrowing”-requirement for translation is classificatory, and hence arbitrary – following Feynman, the translation of SG to QNNs should focus on simulating quantum systems.

Therefore, the recent application of SG to QNNs were centred at developing **Boltzmann Machines** (BM). These are *artificial NNs learning a probability distribution over its inputs* with the Hopfield-model energy function. BM were successful at simulating quantum systems [57, 58] because of their Hopfield-like statistical construction intrinsic for QM.

The recent efforts aim at employing BM to more complex systems – could they simulate a nontrivial set of bodies like highly-entangled systems? The answer is confirmatory: building upon the concepts of quantum SG-simulator (see [59]), Gardas *et al.* showed that the wave function of the transverse quantum Ising model could be represented by a restricted BM [60]. Unsupervised learning gave the ground-state energy. The analogy to SG relies on modelling the pairwise interaction of bodies.

In conclusion, the mentioned publications evidence the successful fulfilment of Requirement 1.

[5] See [53] for other approaches.

[6] **catharsis** [kəˈθar:.sɪs] *noun* – the release of strong emotions [or thoughts] through (...) writing/theatre, helpful in understanding those emotions. Adapted from the *Cambridge Dictionary.*

4.1.2 Requirement 2: Discrete vs continuous theory

Act I showed that the discrete theories have been historically underused. The increase of computing sophistication should make them bloom. However, there is little evidence of higher than speculatory value to support this claim for QNNs. So far, the translation of the SG-framework to QNNs was done primarily via continuous theories [61, 62].

4.1.3 Requirement 3: Do QNNs behave like SG?

Theorising in 4.1.1 and 4.1.2 needs to be supported by experimental evidence for a complete translation. The translation would be of lower value if there were no QNNs resembling SG.

Fortunately, technical realisation of QC is believed to be achievable through **quantum annealing** (QA) [63, 64]. It is *an optimisation technique suitable for a system with many local minima where the time-dependent Schrödinger Equation determines the system evolution*.

In specific, QA could be applied to NNs [65]. For a set of conditions of a laser experiment, a glassy behaviour typical for SG is recovered as shown by Graß *et al.* [66]. Their experiment investigated quantum states with a QA-algorithm by varying the laser detuning in the external magnetic field. The paper fails to show the application of these findings other than with a vague allusion to a version of BM, nonetheless.

A similar study by Harris *et al.* embedding the simulation on simple-cubic lattices on a quantum processing unit concurs with the Graß's work [59]. Thus, the first-generation QC can simulate simple yet nontrivial many-body quantum systems, which is proved by Fig. 11.

4.2 An open question

To address Requirement 4, one needs to learn the ongoing instabilities in the field. This section is a case study of the human brain being a quantum SG object. Originally, the idea was unrelated to SG, having been proposed by Wigner in a 1961 thought experiment [67].

The next section will assess how the SG-framework could settle this discussion.

Firstly, a 1996 Maths-first-Physics-second-style paper by Nishimori and Nonomura doubted

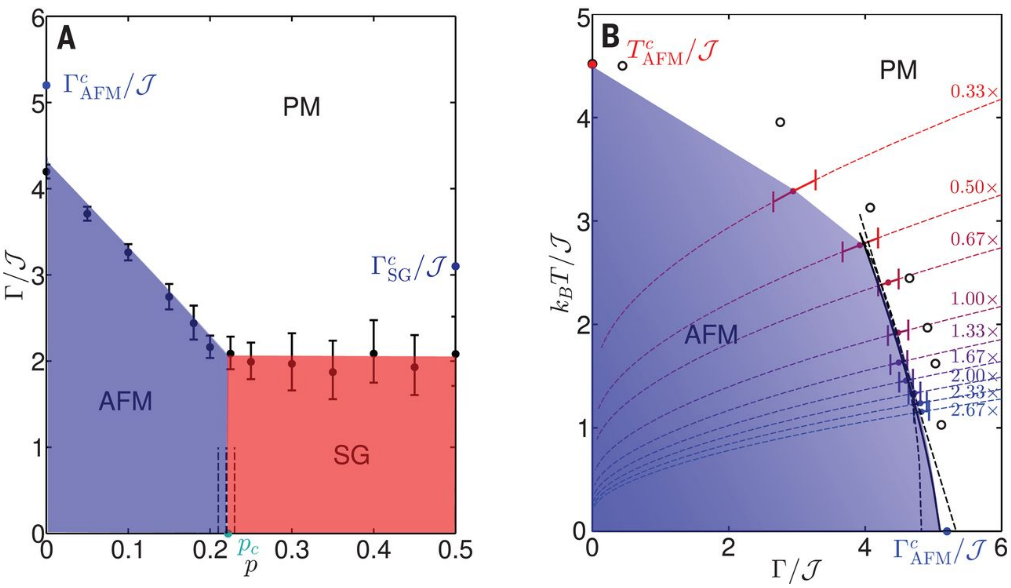


Figure 11: (A) Quantum annealing of correlated electrons yielded this phase diagram. Spin glass, paramagnetic and antiferromagnetic phases were differentiated by Harris *et al.* This was done using the Edwards-Anderson order parameter. (B) It is a graph for a different electrons' coupling potential proving the classical results. From [59].

the quantum origins of real NNs [68]. The authors solved an extension of the Hopfield model with

$$\mathcal{H}_{\rm N\&N} = -\sum_{\substack{\text{all combinations}\\ \text{of neighbours}}} \begin{pmatrix}\text{Interaction magnitude equal to a pairwise}\\ \text{sum of the products of ones and minus}\\ \text{ones ascribed to all the sites}\end{pmatrix}\cdot\begin{pmatrix}\text{Components of the}\\ z\text{−direction Pauli}\\ \text{matrix at site } i\end{pmatrix}\cdot\begin{pmatrix}\text{Components of the}\\ z\text{−direction Pauli}\\ \text{matrix at site } j\end{pmatrix}$$

$$-\,(\text{a parameter})\cdot\sum_{\text{all sites}}\begin{pmatrix}\text{Components of the}\\ x\text{−direction Pauli}\\ \text{matrix at site } i\end{pmatrix} = -\sum_{ij} J_{ij}\sigma_i^z\sigma_j^z - \Delta\sum_i \sigma_i^x, \quad (7)$$

describing quantum oscillations with Pauli matrices. This research was to consider signal losses in synaptic connections as a result of quantum fluctuations.

The SG-system built for the sake of analysis results in a phase diagram similar to that of solely thermally fluctuated system. No additional information was gained. Hence, quantum fluctuations could not explain the macroscopic firing dynamics otherwise than using a classical model with nonzero temperature.

Eleven years later, Perus *et al.* complicated the discussion [69].

They constructed a QNN-algorithm mathematically equivalent to the Hopfield model (in terms of the collective dynamics in neural and quantum complex systems). Thus, the model described biologically realisable systems.

However, they indicated that some information in QNNs is not fully accessible as compared to classical NNs. In short, the system wavefunction would have to be collapsed to get information out of it (cf. [67]). This difference inspired an algorithm with some limited capability for training dataset-based predictions, but not obeying entanglement.

Finally, the "quantumness of mind" was readdressed by the landmark publication for the QNNs-field in 2014 [1]. Schuld *et al.* argued that QM is unlikely to determine the brain's behaviour because of their macroscopic size and dynamics being on the microseconds timescale. For the quantum processes involve millions of ions in a confined space with decoharence in the order of tens of picoseconds [70].

Additionally, [1] set out a three-requirement definition for QNNs corresponding to the Hopfield model and memory effects (i.e., initial state encoded by strings, neural computing mechanism, QM-based evolution). The challenge in reconciling these three remains in connecting linear QM and non-linear activation functions of NNs. In [1], the authors called for an approach different from associating collapse with non-linearity so it does not stipulate a classical evolution.

4.3 Requirement 4: Evidence of progress and prognosis for the future work

To conclude, let the advancements on section 4.2, possibly settling the QMi-debate, be indicated:

- building SG-systems resembling biological systems better,
- improving simulations with SG,
- developing photonics technology.

The first area is well summarised in [71] that reported a system of quantum dots (a physical

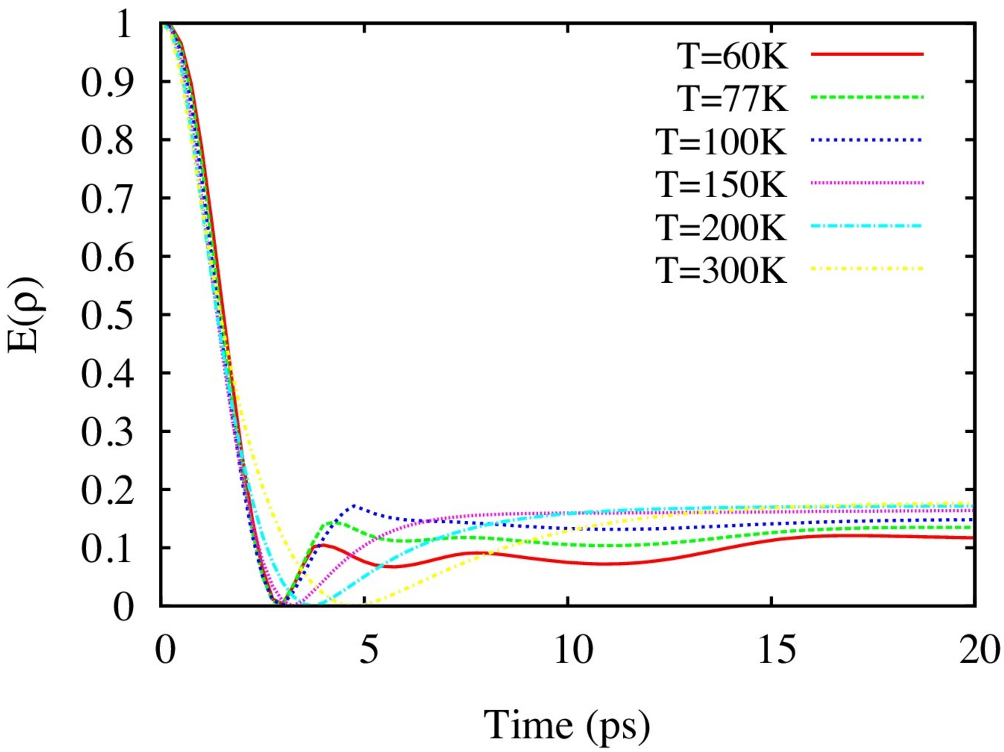


Figure 12: Time dependence of entanglement of formation calculated for the proposed InGaAs/GaAs quantum dot system. Appreciate the close alignment of curves for increasing temperatures. Taken from [71].

representation of QNNs) maintaining coherence at temperatures much higher than counterparts – Fig. 12. Although the authors scrupulously described the mathematical machinery of their simulations, the practicalities (like the ratio of In in GaAs) were sparsely discussed.

The authors claim their model to differ from circuit-based QC, and to reproduce biological systems superiorly to the existing SG-based QNNs. The latter is because the interaction-term in Hamiltonian flips the states of two interacting qubits dynamically (fluctuating environment included), which seems plausible to the author of this review.

Another breakthrough in QMi could stem from improvements in the SG-theory itself. The way it was a catalyst for Perus *et al.*, the improvements in simulating SG could elucidate the research in QNNs.

In this context, the publications by Fan *et al.* [72] and Juenger *et al.* [73] seem to be particularly valuable because of their connections to deep reinforcement learning of NN and ground-state computations on graphs, respectively.

Likewise, [74] seems to open new possibilities as it has influenced the discussion on neural annealing. This is because of the more quantum-mechanical approach to annealing.

Eventually, the public investments into neuroscience [75] suggest that to approve or disapprove QMi, we should look closely at the interplay of SG and photonics.

Interest was aroused in 2019, when artificial NNs were enabled by nanophotonics [76]. Okawachi *et al.* demonstrated a proof of concept for the nanophotonic SG in 2020 [77]. Then-sparked momentum in the field of SG-based optical simulators is proven by publications like [78] and [79]. However, it is the possibility of realising fully functional quantum circuits with photonics that prognoses intense development of QNNs using SG [80].

See Fig. 13

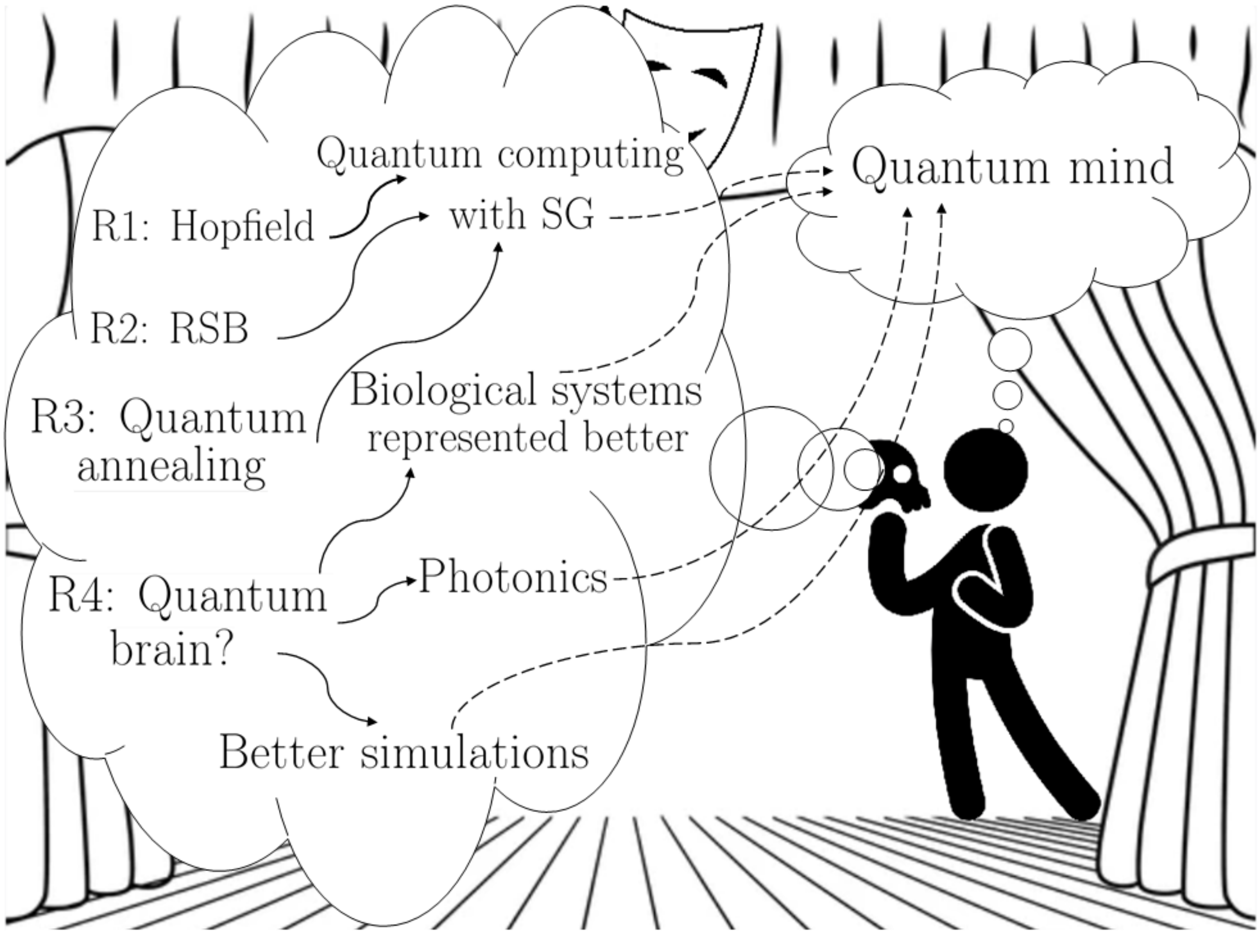


Figure 13: The summary of Act III – the translation of Spin Glass into quantum neural networks may lead to the development of the idea of quantum mind. The future shall show which influence will be the most impactful.

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5 Finale – Conclusion

Shakespearian "To be or not to be?" was answered in positive for the translation of the SG-framework into QNNs. As shown, BMs and other QCs enable simulating quantum systems using both SG and QNNs (Requirement 1). This is with the primary use of the RSB, and because QNNs behave like some SG-systems, e.g., quantum annealers (Requirement 2 & 3).

Importantly, there is much value in pursuing this translation (Requirement 4); section 4.3. explained "why to be?". The answer is that SG has a predictive power to expand the QNN-field as evidenced by SG-models realisable in living organisms, SG-simulations, and photonics.

This statement results from a close look at the interplay of SQ-models (Act I). Their varied applicability to biological, financial, and mathematical structures determined the translational requirements applicable to new applications (Act II). The future work on SG-interdisciplinarity should pinpoint these requirements for other disciplines, e.g., genetics [81], or scanning tunnelling microscopy [82]. The framework presented in this review is hoped to be useful in that research.

For – as remarked by Parisi-critiquing Newman – "this world is of a single piece; yet, we invent nets to trap it for our inspection" [83].

Acknowledgements

Umberto Eco wrote that a scientific writer "must (...) pay debts (...) more difficult to document [than by referencing publications]. It is a good rule of academic honesty to mention (...) a private conversation with a scholar" [84].

The idea for this review would not have been sparked if it had not been for Prof. Steven Bramwell. His boldly asked question if spin glass was indeed worth the 2021 Nobel Prize in Physics a day after its announcement and his continued support throughout this academic year were a driving force for the author.

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